3.1 | Introduction to Integers

Learning Objectives

By the end of this section, you will be able to:

3.1.1 Locate positive and negative numbers on the number line
3.1.2 Order positive and negative numbers
3.1.3 Find opposites
3.1.4 Simplify expressions with absolute value
3.1.5 Translate word phrases to expressions with integers

Be Prepared!

Before you get started, take this readiness quiz.

1. Plot 0, 1, and 3 on a number line.
   
   If you missed this problem, review Example 1.1.
2. Fill in the appropriate symbol: (=, <, or >): 2 ___ 4
   
   If you missed this problem, review Example 2.3.

Locate Positive and Negative Numbers on the Number Line

Do you live in a place that has very cold winters? Have you ever experienced a temperature below zero? If so, you are already familiar with negative numbers. A negative number is a number that is less than 0. Very cold temperatures are measured in degrees below zero and can be described by negative numbers. For example, \(-1{\text{°F}}\) (read as “negative one degree Fahrenheit”) is 1 degree below 0. A minus sign is shown before a number to indicate that it is negative. Figure 3.2 shows \(-20{\text{°F}}\), which is 20 degrees below 0.

Temperatures below zero are described by negative numbers.

Temperatures are not the only negative numbers. A bank overdraft is another example of a negative number. If a person writes a check for more than he has in his account, his balance will be negative.

Elevations can also be represented by negative numbers. The elevation at sea level is 0 feet. Elevations above sea level are positive and elevations below sea level are negative. The elevation of the Dead Sea, which borders Israel and Jordan, is about 1,302 feet below sea level, so the elevation of the Dead Sea can be represented as \(-1,302\) feet. See Figure 3.3.
The surface of the Mediterranean Sea has an elevation of 0 ft. The diagram shows that nearby mountains have higher (positive) elevations whereas the Dead Sea has a lower (negative) elevation.

 Depths below the ocean surface are also described by negative numbers. A submarine, for example, might descend to a depth of 500 feet. Its position would then be −500 feet as labeled in Figure 3.4.

Both positive and negative numbers can be represented on a number line. Recall that the number line created in Section 1.2 started at 0 and showed the counting numbers increasing to the right as shown in Figure 3.5. The counting numbers (1, 2, 3, …) on the number line are all positive. We could write a plus sign, +, before a positive number such as +2 or +3, but it is customary to omit the plus sign and write only the number. If there is no sign, the number is assumed to be positive.

Now we need to extend the number line to include negative numbers. We mark several units to the left of zero, keeping the intervals the same width as those on the positive side. We label the marks with negative numbers, starting with −1 at the first mark to the left of 0, −2 at the next mark, and so on. See Figure 3.6.
The arrows at either end of the line indicate that the number line extends forever in each direction. There is no greatest positive number and there is no smallest negative number. What about zero? Zero is neither positive nor negative.

**MM** Doing the Manipulative Mathematics activity "Number Line-part 2" will help you develop a better understanding of integers.

### Example 3.1

Plot the numbers on a number line:
(a) 3  (b) −3  (c) −2

**Solution**

Draw a number line. Mark 0 in the center and label several units to the left and right.

(a) To plot 3, start at 0 and count three units to the right. Place a point as shown in Figure 3.7.

![Figure 3.7](image)

(b) To plot −3, start at 0 and count three units to the left. Place a point as shown in Figure 3.8.

![Figure 3.8](image)

(c) To plot −2, start at 0 and count two units to the left. Place a point as shown in Figure 3.9.

![Figure 3.9](image)

### 3.1 Plot the numbers on a number line.
(a) 1  (b) −1  (c) −4

### 3.2 Plot the numbers on a number line.
(a) −4  (b) 4  (c) −1

### Order Positive and Negative Numbers

We can use the number line to compare and order positive and negative numbers. Going from left to right, numbers increase in value. Going from right to left, numbers decrease in value. See Figure 3.10.

![Figure 3.10](image)
Just as we did with positive numbers, we can use inequality symbols to show the ordering of positive and negative numbers. Remember that we use the notation \( a < b \) (read \( a \) is less than \( b \)) when \( a \) is to the left of \( b \) on the number line. We write \( a > b \) (read \( a \) is greater than \( b \)) when \( a \) is to the right of \( b \) on the number line. This is shown for the numbers 3 and 5 in Figure 3.11.

![Figure 3.11](image)

The number 3 is to the left of 5 on the number line. So 3 is less than 5, and 5 is greater than 3.

The number lines to follow show a few more examples.

(a)

4 is to the right of 1 on the number line, so 4 > 1.

1 is to the left of 4 on the number line, so 1 < 4.

(b)

−2 is to the left of 1 on the number line, so −2 < 1.

1 is to the right of −2 on the number line, so 1 > −2.

(c)

−1 is to the right of −3 on the number line, so −1 > −3.

−3 is to the left of −1 on the number line, so −3 < −1.

**Example 3.2**

Order each of the following pairs of numbers using < or >:

(a) 14 ___ 6  (b) −1 ___ 9  (c) −1 ___ −4  (d) 2 ___ −20

**Solution**

Begin by plotting the numbers on a number line as shown in Figure 3.12.

![Figure 3.12](image)
(a) Compare 14 and 6.
14 is to the right of 6 on the number line. 14 > 6

(b) Compare −1 and 9.
−1 is to the left of 9 on the number line. −1 < 9

(c) Compare −1 and −4.
−1 is to the right of −4 on the number line. −1 > −4

(d) Compare 2 and −20.
2 is to the right of −20 on the number line. 2 > −20

3.3 Order each of the following pairs of numbers using < or >.
(a) 15___7 (b) −2___5 (c) −3___−7 (d) 5___−17

3.4 Order each of the following pairs of numbers using < or >.
(a) 8___13 (b) 3___−4 (c) −5___−2 (d) 9___−21

Find Opposites

On the number line, the negative numbers are a mirror image of the positive numbers with zero in the middle. Because the numbers 2 and −2 are the same distance from zero, they are called opposites. The opposite of 2 is −2, and the opposite of −2 is 2 as shown in Figure 3.13(a). Similarly, 3 and −3 are opposites as shown in Figure 3.13(b).

![Figure 3.13](image)

**Example 3.3**

The opposite of a number is the number that is the same distance from zero on the number line, but on the opposite side of zero.
Find the opposite of each number:

(a) 7
(b) −10

**Solution**

(a) The number 7 is the same distance from 0 as 7, but on the opposite side of 0. So −7 is the opposite of 7 as shown in Figure 3.14.

(b) The number 10 is the same distance from 0 as −10, but on the opposite side of 0. So 10 is the opposite of −10 as shown in Figure 3.15.

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### Opposite Notation

Just as the same word in English can have different meanings, the same symbol in algebra can have different meanings. The specific meaning becomes clear by looking at how it is used. You have seen the symbol “−”, in three different ways.

- **10 − 4** Between two numbers, the symbol indicates the operation of subtraction.
  - We read 10 − 4 as 10 minus 4.
- **−8** In front of a number, the symbol indicates a negative number.
  - We read −8 as negative eight.
- **−x** In front of a variable or a number, it indicates the opposite. We read −x as the opposite of x.
  - **−(−2)** Here we have two signs. The sign in the parentheses indicates that the number is negative 2. The sign outside the parentheses indicates the opposite. We read −(−2) as the opposite of −2.

By writing a minus sign in front of a number, we can indicate the opposite. So −18 indicates the opposite of 18. Similarly, −(−5) indicates the opposite of −5.
Opposite Notation

$-a$ means the opposite of the number $a$

The notation $-a$ is read the opposite of $a$.

Example 3.4

Simplify: $-(-6)$.

**Solution**

The expression $-(-6)$ has two minus sign symbols. The one inside the parentheses indicates negative 6. The one outside the parentheses indicates the opposite. Together, they indicate the opposite of negative six. The opposite of $-6$ is 6.

3.7 Simplify:

$-(-1)$

3.8 Simplify:

$-(-5)$

Integers

The set of counting numbers, their opposites, and 0 is the set of integers.

Integers

Integers are counting numbers, their opposites, and zero.

$$\{\ldots -3, -2, -1, 0, 1, 2, 3\ldots \}$$

We can use integers to evaluate expressions. Recall that when we evaluate an expression, we substitute a given number for the variable and then simplify. For example, suppose we are given the expression $x$. We can evaluate by substituting different integers for $x$. Given 6, for example, the value of $x$ is 6. Given $-3$, the value of $x$ is $-3$.

We can also evaluate expression to find opposites. Suppose we are given the opposite of $x$, which is shown as $-x$.

Given 6 again, we find $-(6)$, which simplifies to $-6$. Given $-3$ again, we find $-(-3)$, which simplifies to 3. We must be very careful with the signs when evaluating the opposite of a variable.

Example 3.5

Evaluate $-x$: (a) when $x = 8$ and (b) when $x = -8$.

**Solution**
3.9 Evaluate:  
\(-n\): (a) when \(n = 4\)  
(b) when \(n = -4\)

3.10 Evaluate:  
\(-m\): (a) when \(m = 11\)  
(b) when \(m = -11\)

**Simplify Expressions with Absolute Value**

We saw that numbers such as 5 and -5 are opposites because they are the same distance from 0 on the number line. They are both five units from 0. The distance between 0 and any number on the number line is called the absolute value of that number. Because distance is never negative, the absolute value of any number is never negative.

The symbol for absolute value is two vertical lines on either side of a number. So the absolute value of 5 is written as \(|5|\), and the absolute value of -5 is written as \(|-5|\) as shown in Figure 3.16.

**Figure 3.16**

**Absolute Value**

The absolute value of a number is its distance from 0 on the number line.

The absolute value of a number \(n\) is written as \(|n|\).

\(|n| \geq 0\) for all numbers
Example 3.6

Simplify:
(a) \(|3|\)
(b) \(|-44|\)
(c) \(|0|\)

Solution
(a) For \(|3|\), 3 is 3 units from zero. So, \(|3| = 3\).
(b) For \(|-44|\), -44 is 44 units from zero. So, \(|-44| = 44\).
(c) For \(|0|\), 0 is already at 0. So, \(|0| = 0\).

Example 3.7

Evaluate:
(a) \(|x|\) when \(x = -35\)
(b) \(|-y|\) when \(y = -20\)
(c) \(-|u|\) when \(u = 12\)
(d) \(-|p|\) when \(p = -14\)

Solution
We treat absolute value bars just like we treat parentheses in the order of operations. We simplify the expression inside first.
(a) To find \(|x|\) when \(x = -35\):

\[
\text{Substitute } -35 \text{ for } x. \quad |-35| \quad \text{Take the absolute value.} \quad 35
\]
(b) To find \(|-y|\) when \(y = -20\):

\[ |-y| \]

Substitute \(-20\) for \(y\).

\[ |(-20)| = 20 \]

Simplify.

Take the absolute value.

20

(c) To find \(-|u|\) when \(u = 12\):

\[ -|u| \]

Substitute 12 for \(u\).

\[ -(12) = -12 \]

Take the absolute value.

-12

(d) To find \(-|p|\) when \(p = -14\):

\[ -|p| \]

Substitute -14 for \(p\).

\[ -(14) = -14 \]

Take the absolute value.

-14

Notice that the result is negative only when there is a negative sign outside the absolute value symbol.

**Example 3.8**

Fill in \(<\), \(>\), or \(=\) for each of the following:

(a) \(|-5|\)__\:<\>\:\:|\:-5|\)  (b) \(|-y|\)__\:<\>\:\:|\:-y|\)  (c) \(-9\)__\:<\>\:\:|\:-9|\)  (d) \(|-7|\)__\:<\>\:\:|\:-7|\)

**Solution**

To compare two expressions, simplify each one first. Then compare.
(a) Simplify each expression. \(|−5| = 5\) and \(−|−5| = −5\)
Then compare 5 and \(−5\).
5 is greater than \(−5\), so \(|−5| > −|−5|\).
(b) Simplify the absolute-value expression. \(−|−8| = −8\)
Then compare 8 and \(−8\).
8 is greater than \(−8\), so \(8 > −1|−8|\).
(c) Simplify the absolute-value expression. \(−|−9| = −9\)
Then compare \(−9\) and \(−9\).
\(−9\) is equal to \(−9\), so \(−9 = −|−9|\).
(d) Simplify the absolute-value expression. \(−|−7| = −7\)
Then compare \(−7\) and \(−7\).
\(−7\) is equal to \(−7\), so \(−|−7| = −7\).

3.15 Fill in <, >, or = for each of the following:
(a) \(|−9| \text{____} −|−9|\) (b) \(2 \text{____} −|−2|\) (c) \(−8\text{____}−|−8|\) (d) \(−|−5|\text{____}−5\)

3.16 Fill in <, >, or = for each of the following:
(a) \(7\text{____} −|−7|\) (b) \(−|−11|\text{____}−11\) (c) \(−4\text{____}−|−4|\) (d) \(−1\text{____}−|−1|\)

Absolute value bars act like grouping symbols. First simplify inside the absolute value bars as much as possible. Then take the absolute value of the resulting number, and continue with any operations outside the absolute value symbols.

Example 3.9

Simplify: (a) \(|9−3|\)  (b) \(4|−2|\)

Solution
For each expression, follow the order of operations. Begin inside the absolute value symbols just as with parentheses.

(a)
\[
\begin{align*}
\text{Simplify inside the absolute value sign.} & \quad |9−3| \\
\text{Take the absolute value.} & \quad 6 \\
\end{align*}
\]

(b)
\[
\begin{align*}
\text{Take the absolute value.} & \quad 4|−2| \\
\text{Multiply.} & \quad 4 \cdot 2 \\
\end{align*}
\]

3.17 Simplify:
(a) \(|12 − 9|\)  (b) \(3|−6|\)

3.18 Simplify:
(a) \(|27 − 16|\)  (b) \(9|−7|\)
Example 3.10

Simplify: $|8 + 7| - |5 + 6|$.

Solution
For each expression, follow the order of operations. Begin inside the absolute value symbols just as with parentheses.

Simplify inside each absolute value sign. $|15| - |11|
Subtract. $4$

3.19 Simplify:
$|1 + 8| - |2 + 5|$

3.20 Simplify:
$|9 - 5| - |7 - 6|$

Example 3.11

Simplify: $24 - |19 - 3(6 - 2)|$.

Solution
We use the order of operations. Remember to simplify grouping symbols first, so parentheses inside absolute value symbols would be first.

Simplify in the parentheses first. $24 - |19 - 3(4)|$
Multiply $3(4)$. $24 - |19 - 12|$
Subtract inside the absolute value sign. $24 - |7|$
Take the absolute value. $24 - 7$
Subtract. $17$

3.21 Simplify:
$19 - |11 - 4(3 - 1)|$

3.22 Simplify:
$9 - |8 - 4(7 - 5)|$
Translate Word Phrases into Expressions with Integers

Now we can translate word phrases into expressions with integers. Look for words that indicate a negative sign. For example, the word negative in “negative twenty” indicates $-20$. So does the word opposite in “the opposite of $20$.”

**Example 3.12**

Translate each phrase into an expression with integers:

(a) the opposite of positive fourteen
(b) the opposite of $-11$
(c) negative sixteen
(d) two minus negative seven

**Solution**

(a) The phrase the opposite of positive fourteen has the word opposite. Recall that an opposite is the same distance from zero on a number line so it is the same number with the opposite sign. The opposite of positive fourteen is $-(14)$, or $-14$.

(b) The opposite of $-11$ has the word opposite. The same number with the opposite sign is $-(-11)$, or $11$.

(c) The phrase negative sixteen has the word negative. We write a minus sign in front of the number, so $-16$.

(d) The phrase two minus negative seven has the word minus. That indicates subtraction. Begin with $2$ and subtract negative seven, which is $(-7)$. So the expression is $2 - (-7)$.

Notice that we have two signs together, we need to use parentheses. Otherwise, the signs can blend together and make the number difficult to read. So we write $2 - (-7)$ instead of $2 - -7$.

**Example 3.13**

Translate into an expression with integers:

(a) The temperature is $12$ degrees Fahrenheit below zero.

(b) The football team had a gain of $3$ yards.

(c) The elevation of the Dead Sea is $1,302$ feet below sea level.

(d) A checking account is overdrawn by $40$.

As we saw at the start of this section, negative numbers are needed to describe many real-world situations. We’ll look at some more applications of negative numbers in the next example.
Solution

Look for key phrases in each sentence. Then look for words that indicate negative signs. Don’t forget to include units of measurement described in the sentence.

(a) Below zero tells us that 12 is a negative number.
   \(-12°F\)

(b) A gain tells us that 3 is a positive number.
   3 yards

(c) Below zero tells us that 1,302 is a negative number.
   \(-1,302\) feet

(d) Overdrawn tells us that 40 is a negative number.
   \(-$40\)

3.25  Translate into an expression with integers:
The football team had a gain of 5 yards.

3.26  Translate into an expression with integers:
The scuba diver was 30 feet below the surface of the water.

We encourage you to go to Appendix B to take the Self Check for this section.

Access these online resources for additional instruction and practice with integers.

- Introduction to Integers (http://openstaxcollege.org/l/24introinteger)
- Simplifying the Opposites of Negative Integers (http://openstaxcollege.org/l/24neginteger)
- Comparing Absolute Value of Integers (http://openstaxcollege.org/l/24abvalue)
- Comparing Integers Using Inequalities (http://openstaxcollege.org/l/24usinginequal)
Simplify Expressions with Absolute Value

We saw that numbers such as 5 and −5 are opposites because they are the same distance from 0 on the number line. They are both five units from 0. The distance between 0 and any number on the number line is called the absolute value of that number. Because distance is never negative, the absolute value of any number is never negative.

The symbol for absolute value is two vertical lines on either side of a number. So the absolute value of 5 is written as |5|, and the absolute value of −5 is written as |−5| as shown in Figure 3.16.

|n| ≥ 0 for all numbers
Example 3.6

Simplify:
(a) \(|3|\)
(b) \(|-44|\)
(c) \(|0|\)

Solution
(a) For \(|3|\), 3 is 3 units from zero. So, \(|3| = 3\).
(b) For \(|-44|\), -44 is 44 units from zero. So, \(|-44| = 44\).
(c) For \(|0|\), 0 is already at 0. So, \(|0| = 0\).

Try it 3.11  Simplify:
(a) \(|12|\)  (b) \(|-28|\)

Try it 3.12  Simplify:
(a) \(|9|\)  (b) \(|37|\)